OKUCC

OpenFOAM Korea User's Community Conference

CMFD
Computational Multi-Fluid Dynamics Lab.

THE MODELING OF RISING BUBBLE PAIR USING ADAPTIVE MESH REFINEMENT

YuSin Jeong, K. Nandakumar Chandran, ManYeong Ha, JuneKee Min

Presented by,

YuSin Jeong
Master Student
School of Mechanical Engineering
Pusan National University

LSCFD
Large Scale Computational Thermo-Fluid Dynamics Lab.

Date: 09.27.2024

❑ Background

- ➢ Multiphase phenomenas are prevalent in nature (e.g., breaking waves, cavitation, droplet generation) and have been extensively applied in engineering fields.
- ➢ This type of multiphase flow, known for enhancing heat transfer through nucleate boiling, is important in industrial settings but challenging to investigate due to the factors involved.
- ➢ Understanding buoyancy-induced bubbly flow is problematic because it involves mass transfer, bubble interactions, and interfacial processes.

Swarm

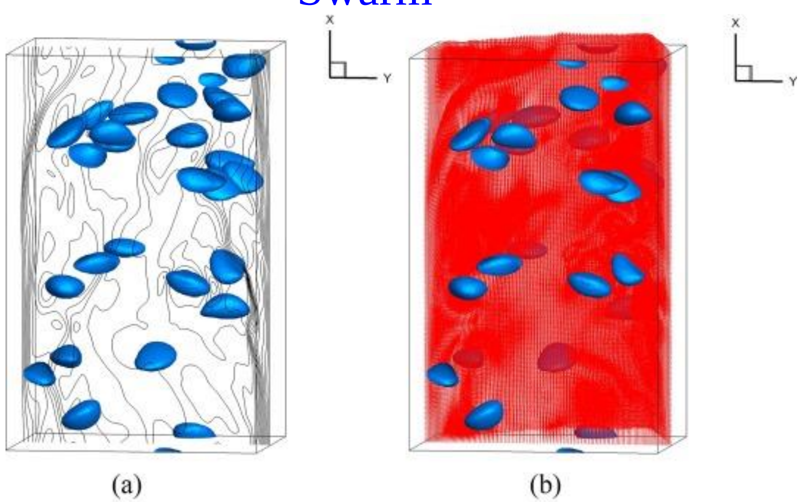


Fig 1. (a) Bubble distribution; (b) Flow field [1]

Bubble pair

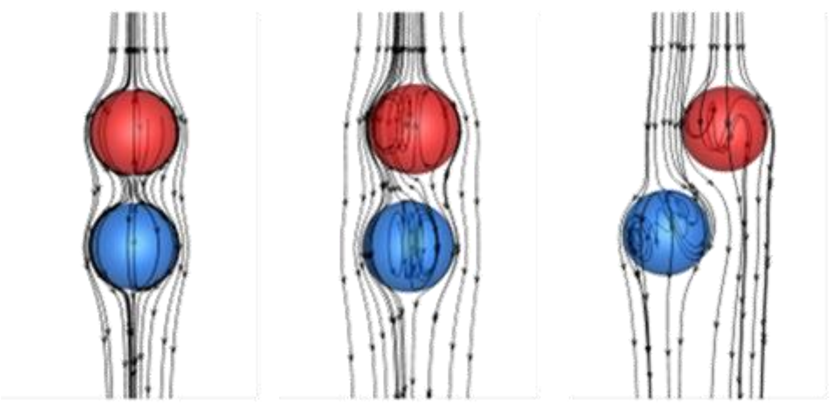


Fig 2. Interaction of bubble pair in quiescent water [2]

MOTIVATIONS

❑ Previous studies

Sl no.	Title	Authors	Brief Description of Work
1	Hydrodynamic Interaction of Two Spherical Bubbles Rising In-Line: A Semi-Analytical Approach, **Chem. Eng. Comm** (2014)	BAZ-RODRI´GUEZ et al.	In the range of moderate to high Reynolds numbers (between 50 and 300), the researchers formulated an equation to describe the axial velocity of a trailing bubble **that is aligned with** a leading bubble.
2	Three-dimensional dynamics of a pair of deformable bubbles rising initially in line. Part 1: Moderately inertial regimes, **J. Fluid Mech** (2020)	Zhang et al.	While focusing on a three-dimensional domain and starting **with an initial in-line configuration**, the researchers examined the impact of both viscous and capillary forces across a broad spectrum of flow conditions.
3	Dynamics of an initially spherical bubble rising in quiescent liquid, **Nat. Comm** (2014)	Tripathi et al.	Utilizing the Galileo number and the Bond number, the researchers identified five distinct regions that show observable **transient deformation of the bubble**.
4	Lift force acting on a pair of clean bubbles rising in-line, **Phys. Fluids** (2019)	Kusuno et al.	In **in-line condition**, they observed lateral migration phenomena **experimentally** at the first time.
5	Wake-induced lateral migration of approaching bubbles, **I. J. Multiphase Flows** (2021)	Kusuno et al.	In **in-line condition**, they explain the bubble's lift reversal movement relating wake and vortex.

❑ RESEARCH GAP

1. Unlike previous studies, which focused on in-lined conditions, we focused on geometrically eccentric configuration.

2. Developing the theoretical model to predict the behavior of the trailing bubble.

❑ Problem Definition

➢ Numerical domain

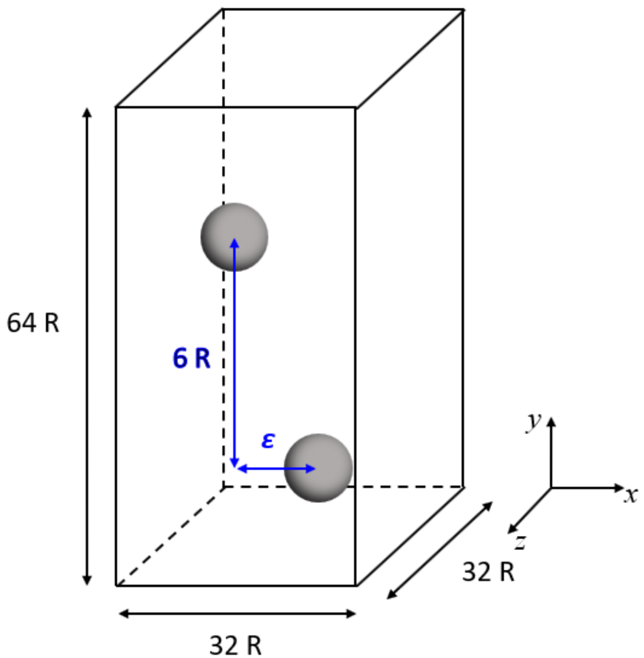


Fig 3. Numerical domain used to solve the GEs

- Initial vertical displacement(S) is fixed as 6 *R*
- To neglect wall effect, enough widths were considered.
- ε, which is the eccentricity between bubbles, is the major parameter of the present study.

➢ Considering bubble shape [4]

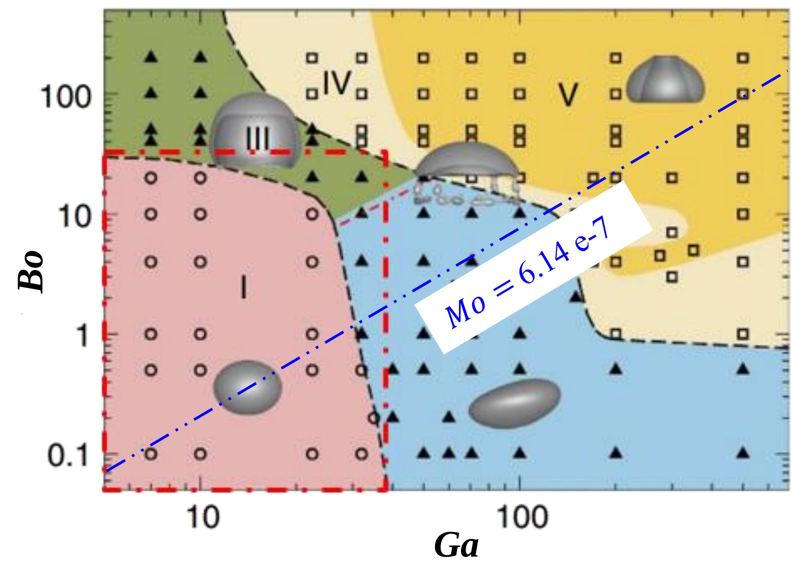


Fig 4. Isolated bubble deformation regime map [3]

- $Ga = \frac{\rho\sqrt{gR}R}{\mu}$, $Bo = \frac{\rho g R^2}{\sigma}$, $Mo = Bo^3/Ga^4$
- To simplify problem, we consider only area where bubble shape can be maintained as sphere.
- To investage the coalescence condition, we studied under range of $Ga < 20$, $Bo < 0.5$

NUMERICAL METHODOLOGY

❑ Brief introduction for InterFoam solver

Assumptions
1. Incompressible
2. Iso-thermal Conditions
3. Laminar Flow Conditions

- Conservation of Mass

$$\nabla . \vec{v} = 0$$

- Conservation of Momentum

$$\underbrace{\frac{\partial \vec{v}}{\partial t} + \nabla . \vec{v}\vec{v}}_{\text{Inertial}} = \underbrace{- \frac{1}{\rho} \nabla P}_{\text{Pressure}} + \underbrace{\frac{1}{\rho} \nabla . \vec{\tau}}_{\text{Viscous}} + \underbrace{\vec{g}}_{\text{Gravity}} + \underbrace{\frac{1}{\rho} \vec{F_s}}_{\text{Surface Tension}}$$

where,

Viscous stress tensor, $\vec{\tau} = \mu \left[\nabla \vec{v} + (\nabla \vec{v})^T\right]$

Surface tension Force, $\vec{F_s} = \sigma \, \mathrm{K} \, \nabla \alpha$ (CSF Model [4])

- Conservation of Volume (VOF)

$$\frac{\partial \alpha}{\partial t} + \nabla . (v\alpha) + \nabla . [\, v_c \alpha \, (1 - \alpha)] = 0$$

$\rho = \rho_l \alpha + \rho_g (1 - \alpha)$,
$\mu = \mu_l \alpha + \mu_g (1 - \alpha)$

- Volume of Fluid (VoF) model [5]

α is volume fraction (Indicator Function)

$$\alpha = \frac{\int_{vol \to \partial x^3} \alpha_l \, dvol}{\int_{vol \to \partial x^3} (\alpha_l + \alpha_g) \, dvol}$$

Where,

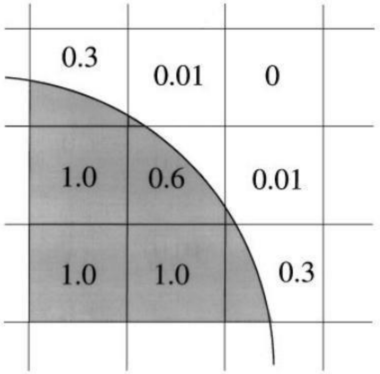
$$v_c = \min[\, C_\alpha \, |\vec{v}| \, , \max(|\vec{v}|)] \frac{\nabla \alpha}{|\nabla \alpha|}$$

Here,

$\alpha = 1$ corresponds to water phase
$\alpha = 0$ corresponds to air phase
$0 < \alpha < 1$ for interface

Estimation of local fluid properties :
A weighted mixture of the physical properties of fluids

$\rho = \rho_l \alpha + \rho_g (1 - \alpha)$,
$\mu = \mu_l \alpha + \mu_g (1 - \alpha)$



(C.Hirt,1981)

NUMERICAL METHODOLOGY

❑ Brief introduction for Adaptive Mesh Refinement (dynamicMeshDict)

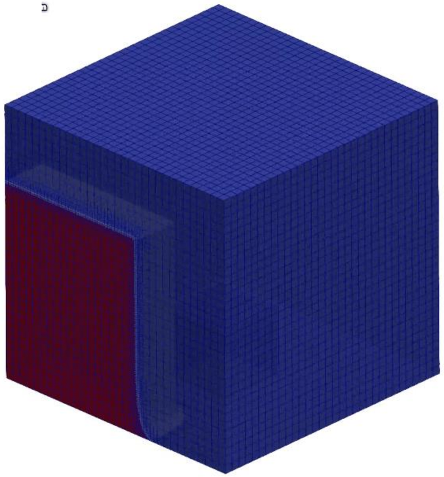


Fig 5. damBreakWithObstacle Tutorial

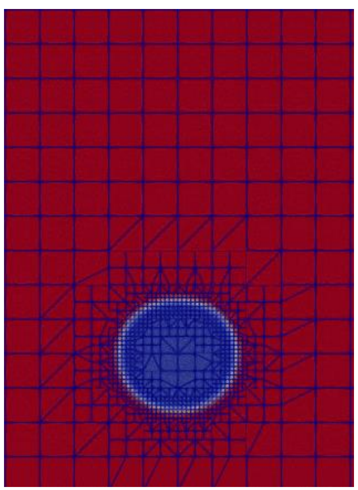


Fig 6. A rising bubble with AMR

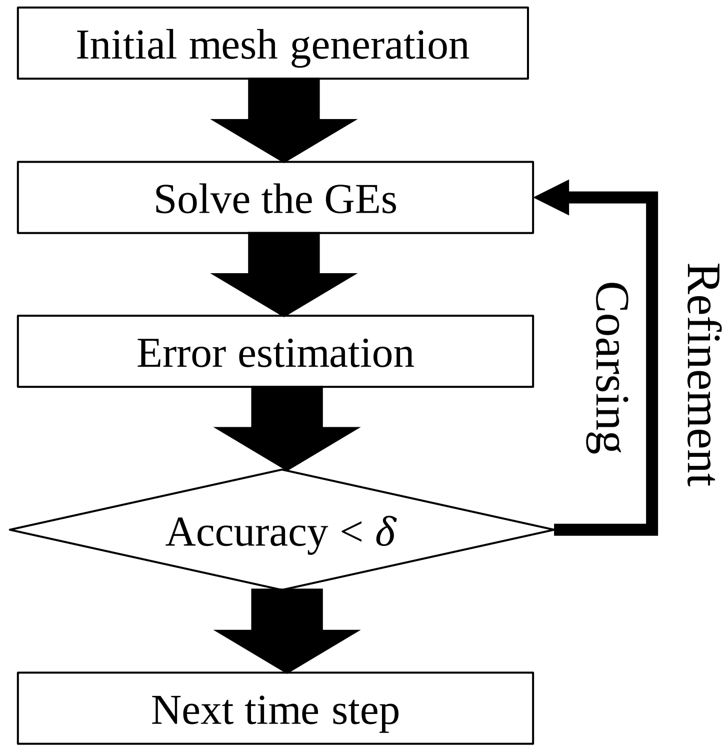


Fig 7. The general algorithm of AMR

- $0.01 < \alpha < 0.9$
- Refeinement Level : 3
- From 2 weeks to 8 hours

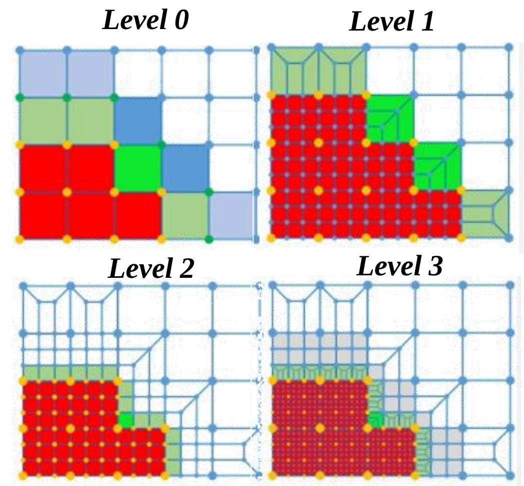


Fig 8. Schematic of AMR levels

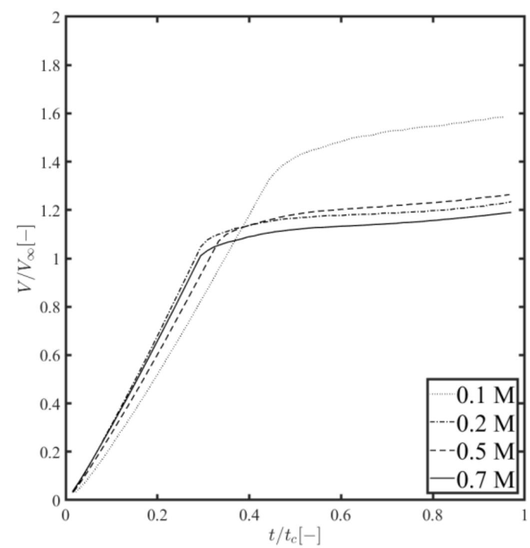


Fig 9. Results of the grid independency study

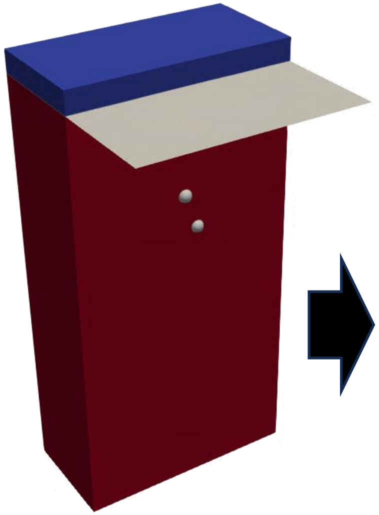
NUMERICAL METHODOLOGY

❑ Postprocessing with image processing technique

From Eulerian data to Lagrangian data

Calculating centroids of bubbles & Aspect ratios

Repeat in each timestep

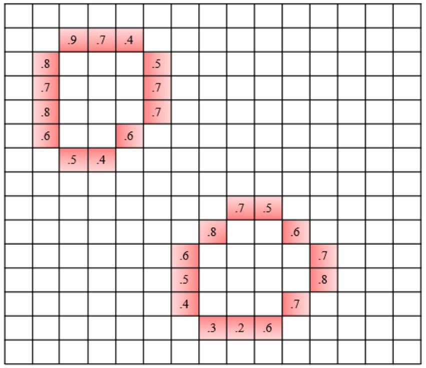


Law data with OpenFOAM

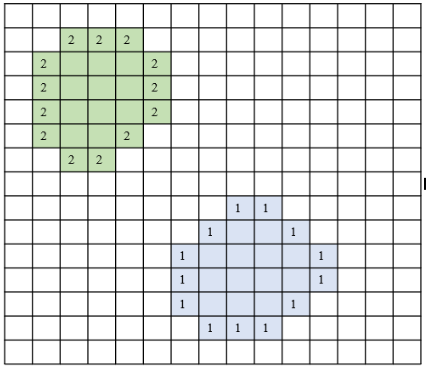
interface_data_0.csv
interface_data_1.csv
interface_data_2.csv
interface_data_3.csv
interface_data_4.csv
interface_data_5.csv
interface_data_6.csv
interface_data_7.csv
interface_data_8.csv
interface_data_9.csv
interface_data_10.csv

MATLAB®

Exporting to Matlab with .csv form



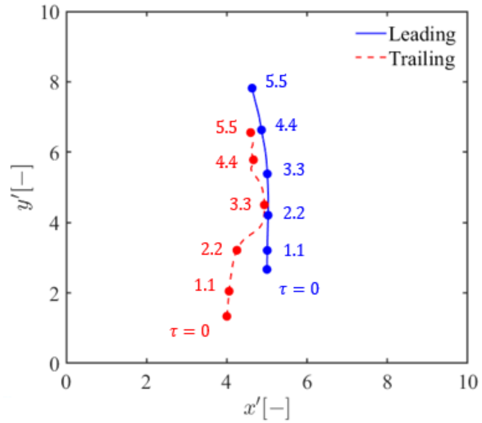
Regrouping

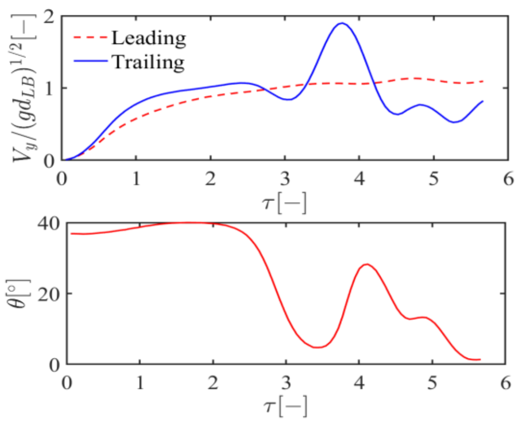


Identifying two bubbles

Brief results

Bubbles position
Bubbles velocity
Angle between bubbles
Drag & Lift forces





❑ Inherent problem of VOF & AMR

AMR working procedure with VOF

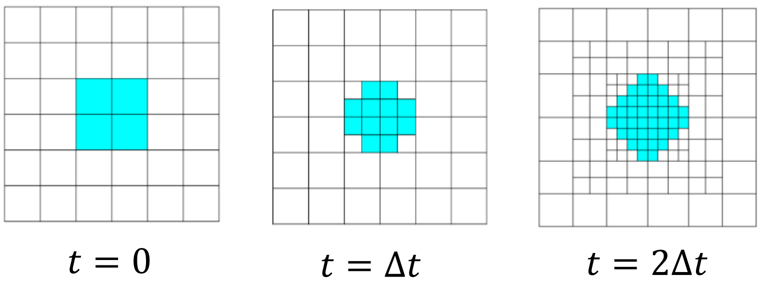


Fig 10. The general procedure of mesh refinement in AMR

Problems in initialization

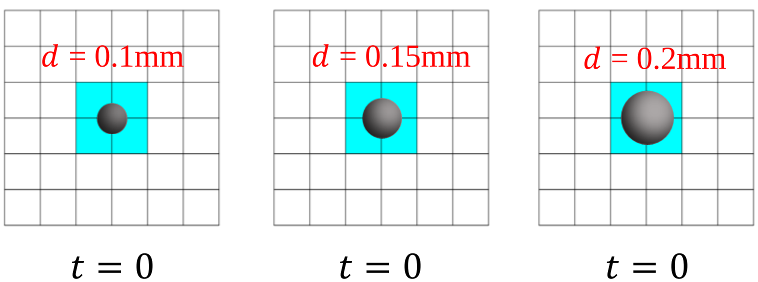


Fig 11. Problem identification when AMR was coupled with VOF

Target bubble sizes & initial AMR grid should be well-matched.

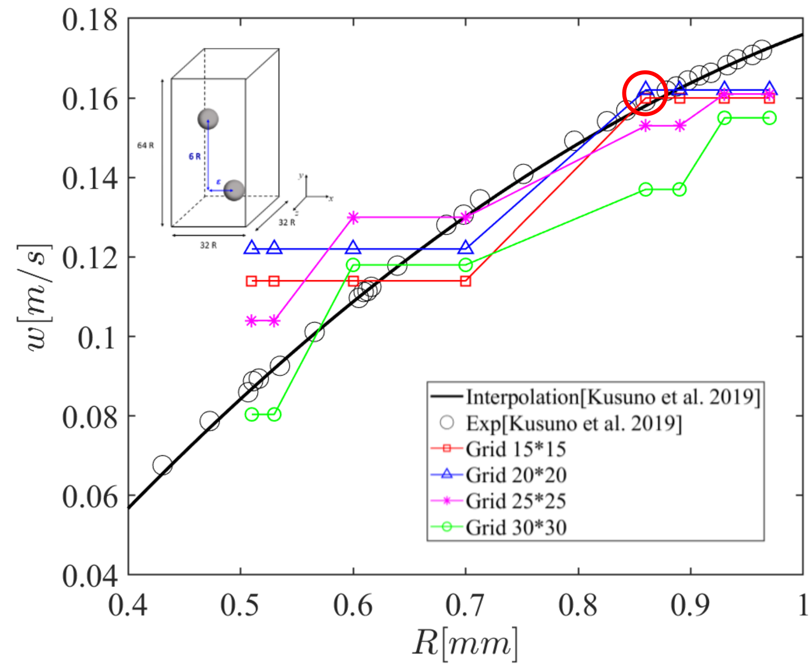


Fig 12. The single bubble terminal velocity from Kusuno et al. [5]

1. CFD shows step-like results
2. Secure the ratio between the bubble and initial grids in fewer grids as much as possible
3. Applying this ratio to the other cases by changing the domain sizes

NUMERICAL METHODOLOGY

❑ Single bubble validation

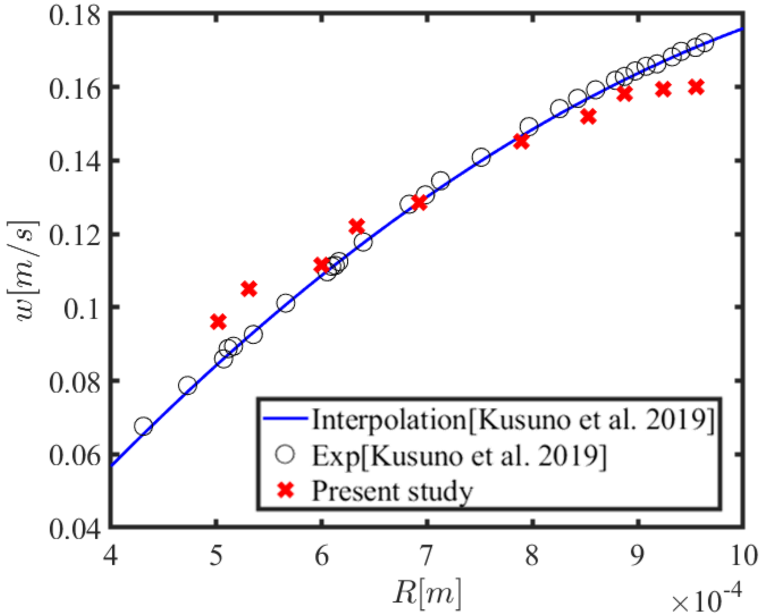
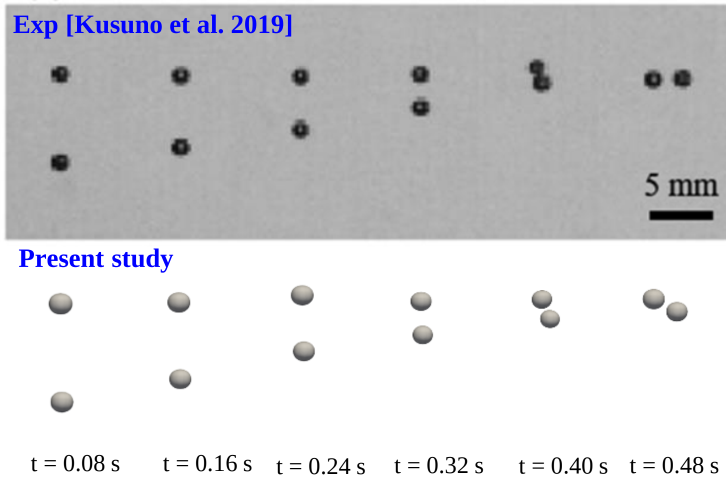


Fig 13. The single bubble validation compared with Kusuno et al.[6]

- To validate the capability of numerical schemes, validation was done about the terminal velocity of a single bubble.
- The single bubble results are well matched to the experimental result from the previous study [Kusuno et al. 2019]
- Trivial errors could originate from the process of bubble generation.

❑ Bubble pair validation [Kusuno et al. 2019]



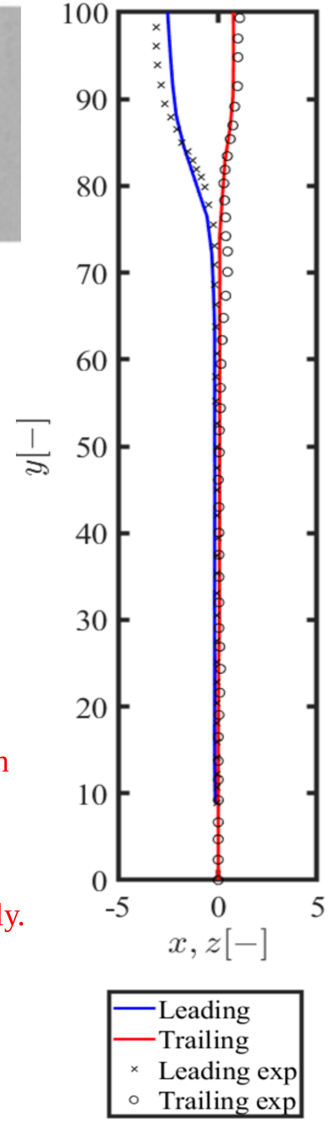
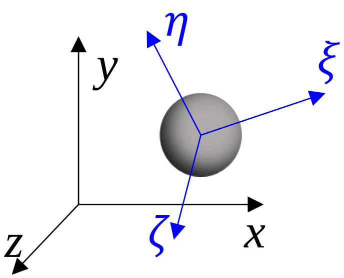


Fig 14. The bubble pair validation with Kusuno et al.[6]

- To verify the capability of capturing the interaction physics between two bubbles.
- Our numerical results are well matched to experimental results quantitatively and qualitatively.

❑ Kirchhoff equation(Purpose of calculating Drag/Lift)

➢ Dynamic Reference Coordinate [7]



Basic concept: Expression of movement of a rigid body in flexible coordinate

Advantage: Particularly in fluid dynamics, convenience to express **added mass force**

Major usage: Analysis of the movement of particles in quiescent fluid

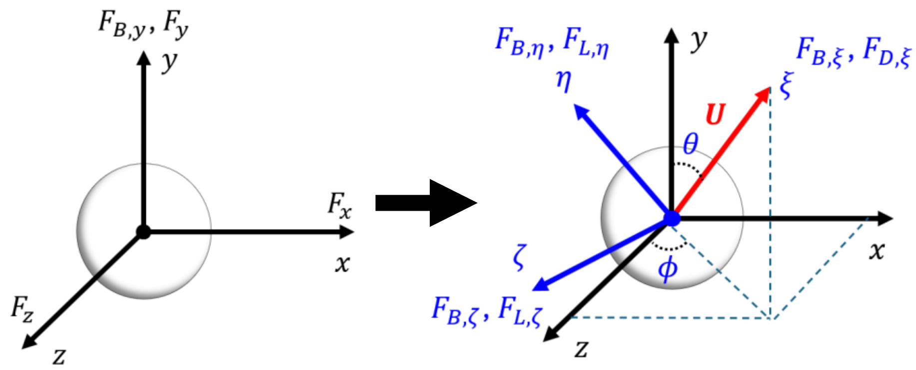


Fig 15. The dynamic reference which is adjusted by the direction of bubble.

Equation of motion in each direction

$$i=\xi \qquad (\mathbb{A}_{\xi\xi}+\cancelto{\text{neglible}}{m_{\xi\xi}})\frac{dU_\xi}{dt}-(\mathbb{A}_{\eta\eta}+\cancelto{\text{neglible}}{m_{\eta\eta}})\cancelto{U_\eta=0}{U_\eta}\Omega_\zeta+(\mathbb{A}_{\zeta\zeta}+\cancelto{\text{neglible}}{m_{\zeta\zeta}})\cancelto{U_\zeta=0}{U_\zeta}\Omega_\eta=F_\xi$$

$$i=\eta \qquad (\cancelto{\mathbb{A}_{\eta\eta}=0}{\mathbb{A}_{\eta\eta}}+\cancelto{\text{neglible}}{m_{\eta\eta}})\frac{dU_\eta}{dt}+(\mathbb{A}_{\xi\xi}+\cancelto{\text{neglible}}{m_{\xi\xi}})U_\xi\Omega_\zeta-(\mathbb{A}_{\zeta\zeta}+\cancelto{\text{neglible}}{m_{\zeta\zeta}})\cancelto{U_\zeta=0}{U_\zeta}\Omega_\xi=F_\eta$$

$$i=\zeta \qquad (\cancelto{\mathbb{A}_{\eta\eta}=0}{\mathbb{A}_{\zeta\zeta}}+\cancelto{\text{neglible}}{m_{\zeta\zeta}})\frac{dU_\zeta}{dt}-(\mathbb{A}_{\xi\xi}+\cancelto{\text{neglible}}{m_{\xi\xi}})U_\xi\Omega_\eta+(\mathbb{A}_{\eta\eta}+\cancelto{\text{neglible}}{m_{\eta\eta}})\cancelto{U_\eta=0}{U_\eta}\Omega_\xi=F_\zeta$$

Major assumptions

1. Bubble is sphere
2. The mass of bubbles can be negligible.
3. Bubble always moves on ξ direction

$$i=\xi \qquad (\mathbb{A}_{\xi\xi})\frac{dU_\xi}{dt}=F_{\xi,B}+F_{\xi,D}$$

$$i=\eta \qquad (\mathbb{A}_{\xi\xi})U_\xi\Omega_\zeta=F_{\eta,B}+F_{\eta,L}$$

$$i=\zeta \qquad -(\mathbb{A}_{\xi\xi})U_\xi\Omega_\eta=F_{\zeta,B}+F_{\zeta,L}$$

Assumtion for bubble shape : $\chi=1$

(Added mass) $\mathbb{A}_{\xi,\xi}=C_m\rho V,\ C_m=0.62\,\chi-0.12$ [8], [9]

Assumtion for simplicity : $\zeta-axis$ is always placed on *xz* plane

Rotation rate of y-axis

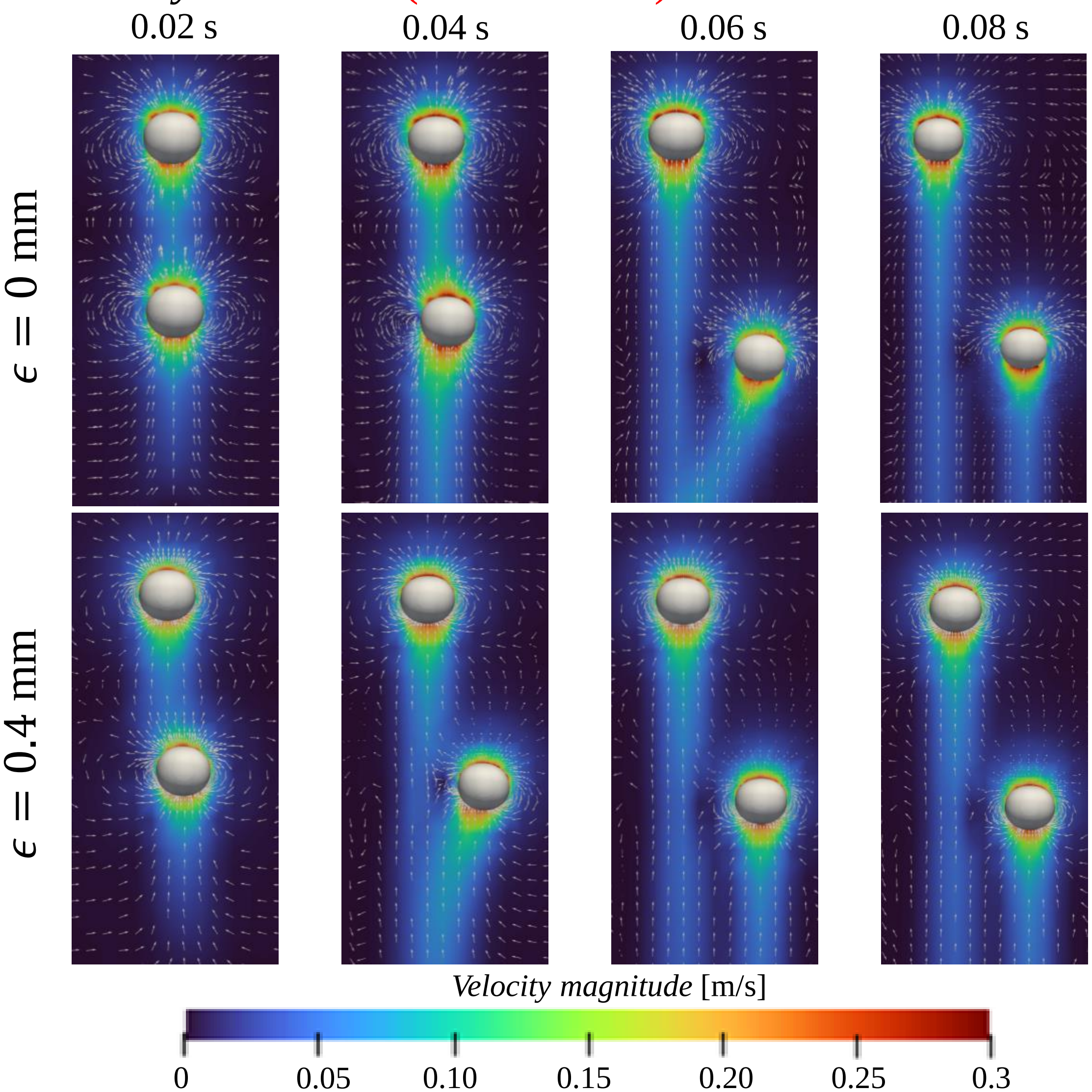
(Rotation rate for ξ) $\Omega_\xi=\frac{d\phi}{dt}cos\theta$ (Rotation rate for ζ) $\Omega_\zeta=-\frac{d\theta}{dt}$

(Rotation rate for η) $\Omega_\eta=\frac{d\phi}{dt}sin\theta$ $\quad C_D=\frac{F_D}{\frac{1}{2}\rho A U_\infty^2}$ $\quad C_L=\frac{F_L}{\frac{1}{2}\rho A U_\infty^2}$

RESULTS AND DISCUSSIONS

❑ Dynamics in pure water ($Mo = g\mu^4(\rho_l - \rho_g)/\sigma^3\rho_l^2$ = 2.86 e-11)

Velocity Contours ($R = 0.4$ mm)



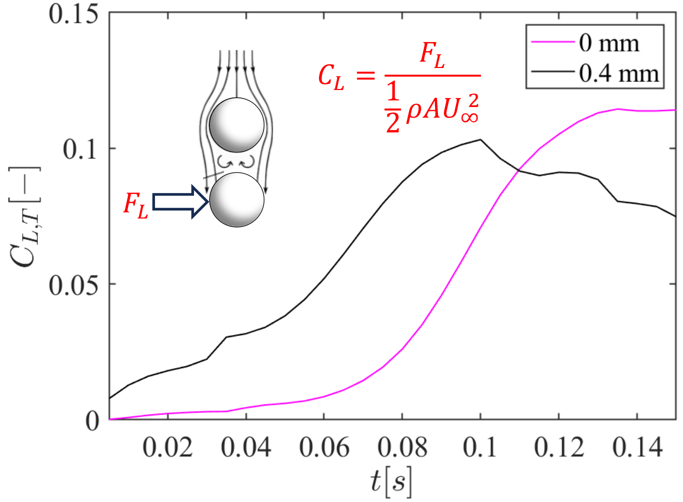


Fig 16. The lift force comparison between two initial eccentricities

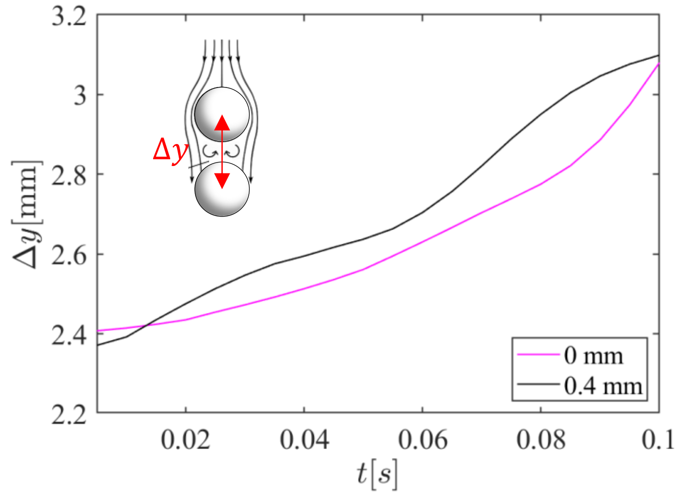


Fig 17. The vertical distance between two initial eccentiricities

RESULTS AND DISCUSSIONS

❑ Lagrangian perspective dynamics modeling

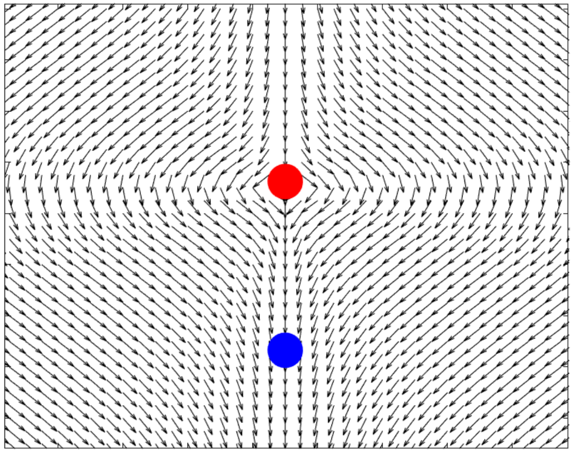


Fig 18. Bubble pairs inside of oseen's flow (Red : LB, Blue : TB)

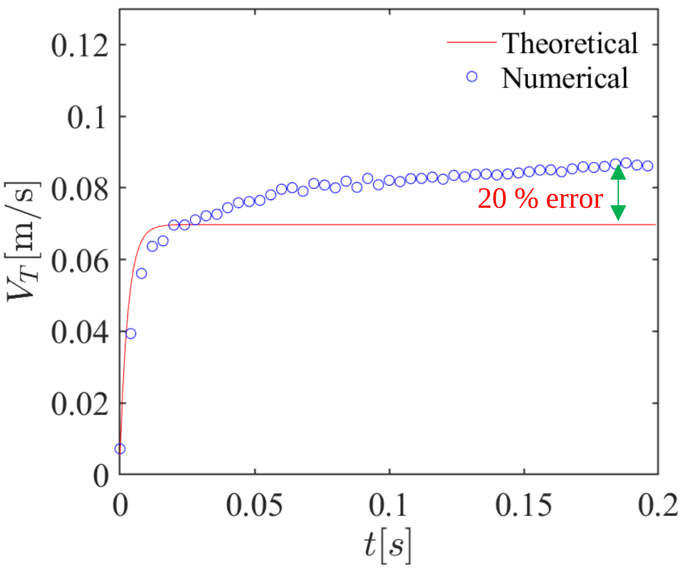


Fig 19. Comparison between the numerical and theoretical results

- Flow field modeling : Oseen's flow

r-dir : $U[-\frac{a^3}{2r^3}cos\theta + \frac{3a^2}{Rer^2} - \frac{3a^2}{Re}\left(\frac{1}{r^2} + \frac{k}{r}[1 - cos\theta]\right)e^{-kr(1+cos\theta)}]$

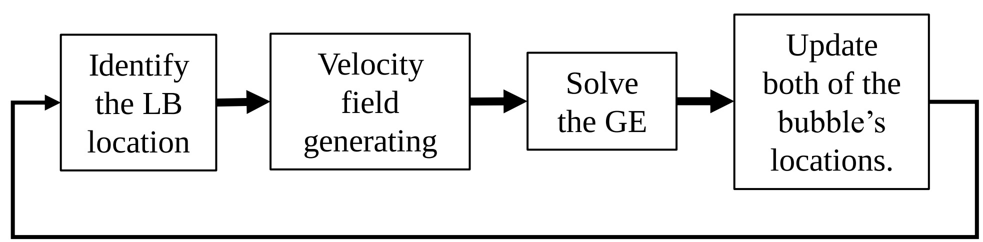
θ-dir : $-U[\frac{a^3}{4r^3}sin\theta + \frac{3a^2}{Rer}ksin\theta e^{-kr(1+cos\theta)}]$

- Governing equation [10]

$St \approx 0.026 \ll 1 \quad Re \approx 0.5 \ll 1$

$$\underbrace{(\rho_f - \rho_d)\frac{4}{3}\pi r_D^3 g}_{\text{Buoyancy}} + \underbrace{3m_{ad}^B\frac{DU_v^B}{Dt}}_{\text{Disturbance modeling (Maxey – Riley eq)}} - \underbrace{6\pi\mu_f r_D(U_D - U_v^B)}_{\text{Drag force (Stokes law)}} = \underbrace{(m + m_{ad})\frac{dU_D}{dt}}_{\text{Inertial term \& Added mass}}$$

- Algorithms for how to model the bubble's movement



Iterative process

CONCLUSION

SUMMARY :

- Dynamics of bubbles (***Ga*** < 20, ***Bo*** < 0.5) were studied numerically.
- Numerical results are validated against the reported experiment both qualitatively and quantitatively.
- Using the Kirchhoff equation, the lift forces have been tracked reversely.
- In the pure water column, the C_L and Δy were investigated.
- The movement of trailing bubble have been modeled using oseen's flow, however, it shows 20 % from the numerical observation.

HOME TAKE MESSAGES :

- In the certain scenario, the AMR technique can reduce computational cost drastically.
- With AMR, the initial grid has to be determined carefully.
- Drag / Lift force can be estimated by the rigid body's velocity.

REFERENCES

1) Feng, Xin, et al. "Bubble induced turbulence model improved by direct numerical simulation of bubbly flow." Chemical Engineering Journal 377 **(2019): 120001.**

2) Zhang, J., Ni, M.J. and Magnaudet, J., 2021. Three-dimensional dynamics of a pair of deformable bubbles rising initially in line. Part 1. Moderately inertial regimes. *Journal of Fluid Mechanics*, ***920*, p.A16**.

3) Tripathi, M.K., Sahu, K.C. and Govindarajan, R., 2015. Dynamics of an initially spherical bubble rising in quiescent liquid. *Nature communications*, ***6*(1), p.6268**.

4) Hirt, C., and B. Nichols. 1981. "Volume of Fluid (Vof) Method for the Dynamics of Free Boundaries." *Journal of Computational Physics* **39 (1): 201–225**

5) Brackbill, J. U., D. B. Kothe, and C. Zemach. 1992. "A Continuum Method for Modeling Surface Tension." *Journal of Computational Physics* **100 (2): 335–354**

6) Kusuno, Hiroaki, Hiroya Yamamoto, and Toshiyuki Sanada. "Lift force acting on a pair of clean bubbles rising in-line." *Physics of Fluids* 31.7 (2019).

7) Zenit, R., Federle, M.E., Das, A. and Mathai, V., The Interaction of an Oil Droplet and a Gas Bubble Rising in a Quiescent Fluid. *Available at SSRN 4524928*.

8) Krishna, R. and Van Baten, J.M., 2001. Simulating the rise characteristics of gas bubbles in liquids using CFD. *Chemical Engineering & Technology: Industrial Chemistry-Plant Equipment-Process Engineering-Biotechnology*, ***24*(4), pp.427-430**.

9) Wu, M. and Gharib, M., 2002. Experimental studies on the shape and path of small air bubbles rising in clean water. *Physics of fluids*, *14*(7), **pp.L49-L52.**

10) Federle, Madeline E., et al. "The interaction of an oil droplet and a gas bubble rising in a quiescent fluid." *International Journal of Multiphase Flow* **171 (2024): 104682.**