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bouyantBoussinesqSimpleFoam 기반 위상최적화 연구

Topology optimization study based on bouyantBoussinesqSimpleFoam

Jae Sung Yang*

Sang Don Lee**
June Kee Min***

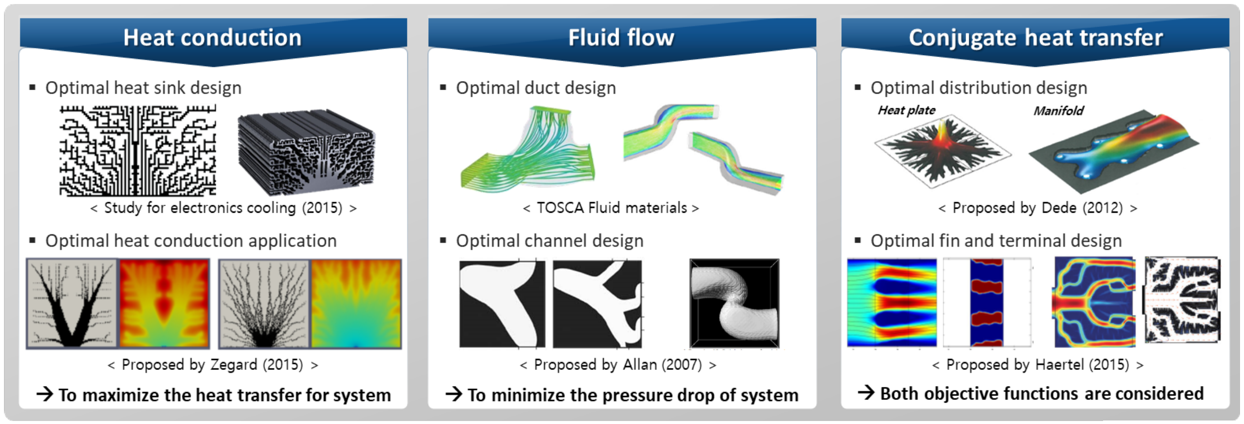
* Rolls-Royce UTC, Pusan National University
** NextFOAM
*** School of Mechanical Engineering, Pusan National University

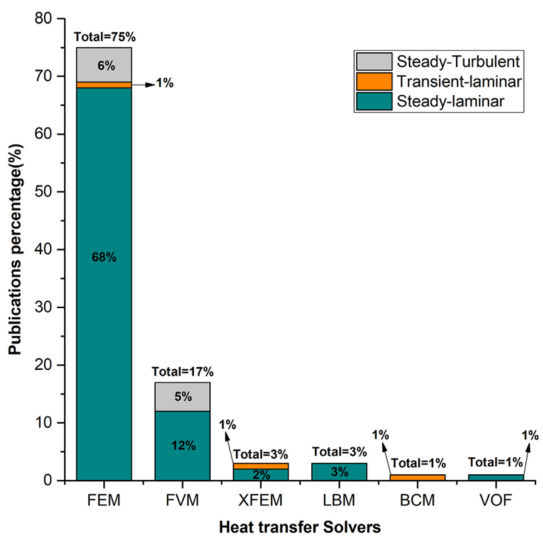
본 연구성과는 2024년도 정부(교육부)의 재원으로 한국연구재단의 지원을 받아 수행된 기초연구사업임(No. RS-2024-00449088)
This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (No. RS-2024-00449088)

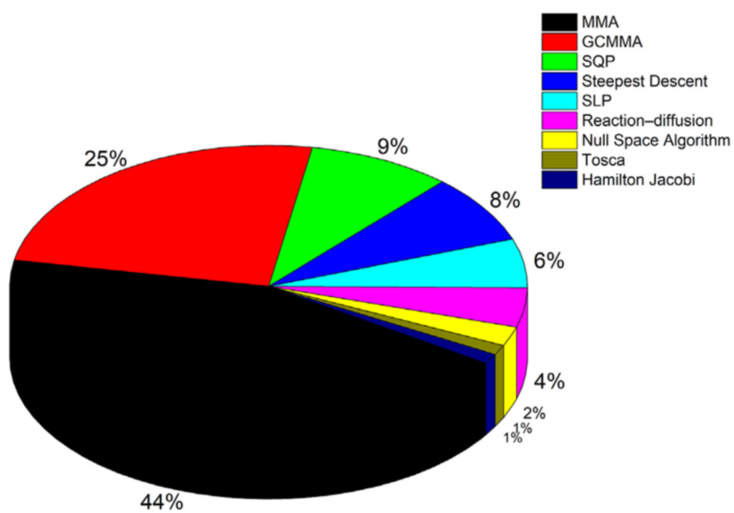
Introduction

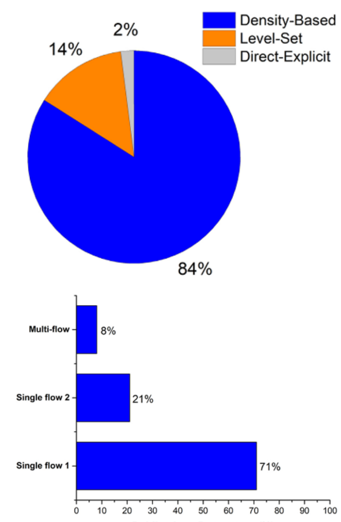
❖ Topology optimization

- 75% of previous topology optimization research for thermal-fluidic problems is based on finite element method.
- FVM based thermal fluidic topology optimization solver development considering continuous adjoint method.









Introduction

❖ *buoyantBoussinesqSimpleFoam*

- **Governing equations**

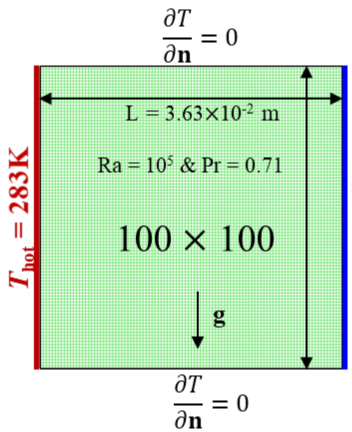
$$-\nabla\cdot\mathbf{u}=0$$

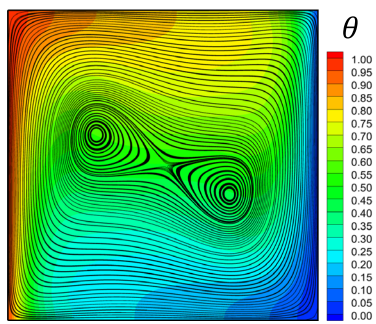
$$(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla\hat{p}_{rgh}-\nabla\cdot[2\nu D(\mathbf{u})]-\mathrm{g}\rho_k=0$$

$$\mathbf{u}\cdot\nabla T-\nabla\cdot[\mathcal{K}\nabla T]=0$$

- **Boussinesq approximation**

$$\rho=\rho_0[1-\beta(T-T_0)]$$





❖ *adjointShapeOptimizationFoam**

- **Governing equations**

Primal

$$\begin{cases}-\nabla\cdot\mathbf{u}=0\\(\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p-\nabla\cdot[2\nu D(\mathbf{u})]+\alpha\mathbf{u}=0\end{cases}$$

Adjoint

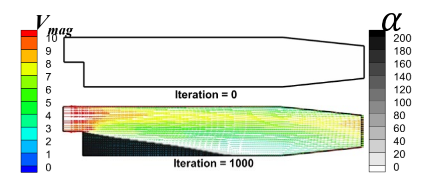
$$\begin{cases}\nabla\cdot\mathbf{v}=0\\\nabla q-2D(\mathbf{v})\mathbf{u}-\nabla\cdot[2\nu D(\mathbf{v})]+\alpha\mathbf{v}=0\end{cases}$$

- **Objective**

$$J_{Dis.}=-\int_{\Gamma}d\Gamma\left(p+{}^{1}/_{2}\,u^2\right)\mathbf{u}\cdot\mathbf{n}\qquad J_{uni.}=-\int_{out}d\Gamma\,{}^{c}/_{2}\left(\mathbf{u}-\mathbf{u}^d\right)^2$$

- **Sensitivity**

$$\frac{\partial\mathcal{L}}{\partial\alpha_i}=\mathbf{u}_i\cdot\mathbf{v}_iV_i$$



- **Optimizer (Steepest descent method)**

$$\alpha_n=\alpha_0-\lambda\frac{\partial\mathcal{L}}{\partial\alpha_i}$$

Topology optimization solver considering Boussinesq approximation
for natural convection and heat transfer studies is developed
coupling *buoyantBoussinsesqSimpleFoam* with *adjointShapeOptimizationFoam*.

CMFD Computational Multi-Fluid Dynamics Lab.

* C. Othmer et al., "Implementation of a continuous adjoint for topology optimization of ducted flows", AIAA, 2007

Numerical methodologies

❖ Continuous adjoint method

- **Objective function**
  - Minimize the difference between the temp. distribution and the desired temperature in the computational domain.

$$J = \frac{1}{2}\int_{\Omega}(T - T_d)^2 d\Omega \qquad (T_d\text{: Desired temperature})$$

  - Augmented objective function, $\mathcal{L}$ (Lagrangian)

$$\mathcal{L} = J + \sum_{i=1}^{n}\int_{\Omega}\lambda_i \Re_i d\Omega = J + \int_{\Omega}\mathbf{v}\Re_{\mathbf{u}} d\Omega + \int_{\Omega} q\Re_p d\Omega + \int_{\Omega} T_a \Re_T d\Omega$$

- **Governing equations (Residuals)**

$$\Re_p = -\nabla\cdot\mathbf{u} \cong 0$$

$$\Re_{\mathbf{u}} = (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla\hat{p}_{rgh} - \nabla\cdot\left(2\nu D(\mathbf{u})\right) - \mathrm{g}\rho_k + \alpha(\gamma)\mathbf{u} \cong 0$$

$$\Re_T = \mathbf{u}\cdot\nabla T - \nabla\cdot(\mathcal{K}(\gamma)\nabla T) \cong 0$$

Derived adjoint equations

$$\Re_q = -\nabla\cdot\mathbf{v} \cong 0$$

$$\Re_{\mathbf{v}} = -(\nabla\mathbf{v})\mathbf{u} - (\mathbf{u}\cdot\nabla)\mathbf{v} + \nabla q - \nabla\cdot\left(2\nu D(\mathbf{v})\right) + \alpha(\gamma)\mathbf{v} + T_a\nabla T \cong 0$$

$$\Re_{T_a} = \mathbf{u}\cdot\nabla T_a - \nabla\cdot(\mathcal{K}(\gamma)\nabla T_a) + \beta\mathbf{v}\cdot\mathrm{g} + (T - T_{Obj}) \cong 0$$

- **Sensitivity**

$$\frac{\partial\mathcal{L}}{\partial\gamma} = \left[\mathbf{u}\cdot\mathbf{v}\frac{\partial\alpha}{\partial\gamma} + \nabla T_a\cdot\nabla T\frac{\partial\mathcal{K}}{\partial\gamma}\right]V_{cell,i}$$

Derived sensitivity

	Primal	Adjoint
Velocity	$\mathbf{u}$	$\mathbf{v}$
Pressure	p	q
Temperature	T	T_a

Numerical methodologies

❖ Numerical method for topology optimization (1/2)

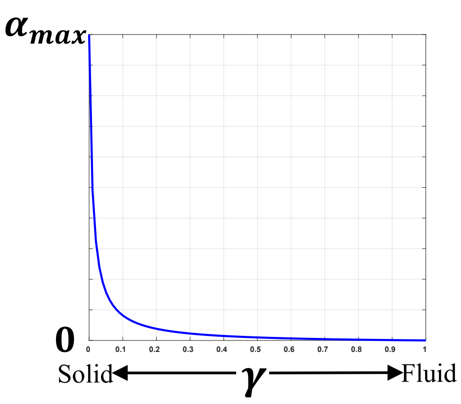
- **Design variable**

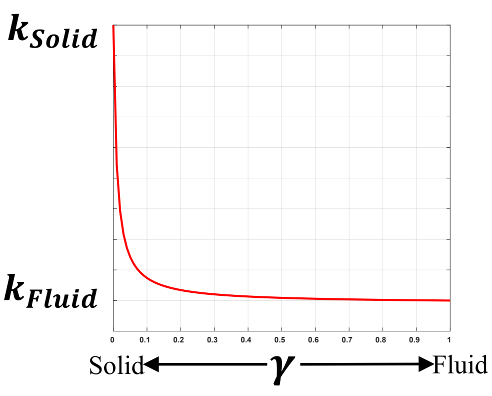
$$0 \le \gamma \le 1 \quad \begin{cases} \gamma = 0: Solid \\ \gamma = 1: Fluid \end{cases}$$

- **Solid Isotropic Material with Penalization (SIMP)**
  - The **inverse permeability** and **thermal conductivity** for the design variable are represented by the SIMP function.

$$\alpha(\gamma) = \alpha_{max} n \frac{1-\gamma}{n+\gamma}$$

$$k(\gamma) = k_{Solid} + (k_{Fluid} - k_{Solid})\gamma \frac{n+1}{n+\gamma}$$





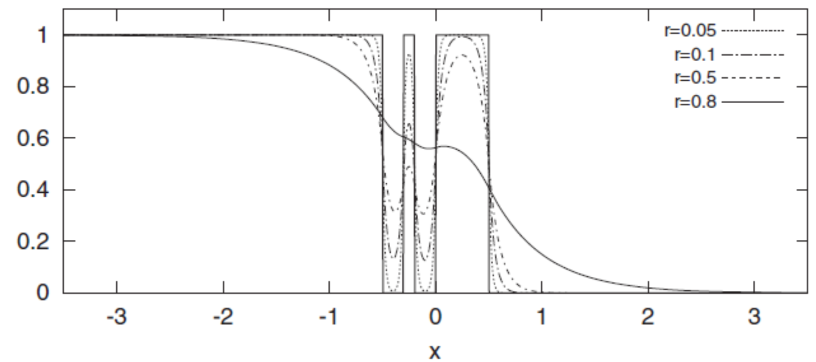
- **Optimizer: OC-algorithm***
  - The optimality criteria (OC) algorithm is implemented to find the optimal design variable distribution controlling step size for volume constraints.

*O.Sigmund, “A 99 line topology optimization code written in Matlab”, Struct Multidisc Optim, 2001

Numerical methodologies

❖ Numerical method for topology optimization (2/2)

- **Helmholtz PDE filter***
  - The Helmholtz partial differential equation is solved for sensitivity and design variables to get a stable solution.
  - Homogeneous Neumann boundary conditions
  - Low value of the length parameter
    - * Capture the detailed shape
    - * Unstable
  - High value of length parameter
    - * Removed small detail
    - * Stable



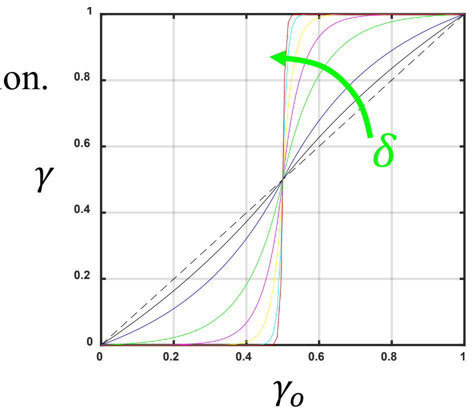
$$-R_f \nabla^2 \phi + \phi = \phi_0 \qquad \frac{\partial \phi}{\partial \mathbf{n}} = 0$$

Length parameter

- **Variable Heaviside step function**
  - The Heaviside step function makes the geometry sharp via the design variable projection.
  - The value of step coefficient(δ) controls the projection sharpness.

$$\gamma_i = 0.5\left[\exp\{-\delta(1-2\gamma_{o,i})\} - (1-2\gamma_{o,i})\exp(-\delta)\right] \qquad [\gamma_i \le 0.5]$$

$$\gamma_i = 0.5 + 0.5\left[1 - \exp\left\{-\delta\left(\frac{\gamma_{o,i}-0.5}{0.5}\right)\right\} + (\gamma_{o,i}-0.5)\frac{\exp(-\delta)}{0.5}\right] \qquad [\gamma_i > 0.5]$$



*Lazarov et al., "Filters in topology optimization based on Helmholtz-type differential equations," *Int. J. Numer Methods Eng.*, 86, pp.765-781, (2011).

Numerical methodologies

❖ Algorithm structure (OpenFOAM ESI v2212)

1. Initialization, $\boldsymbol{\gamma} = 1$ (Fluid)
2. Update inverse permeability, $\boldsymbol{\alpha}$
3. Update thermal conductivity, $\boldsymbol{k}$
4. Evaluate objective, $\boldsymbol{J}$
5. **Primal** pressure-velocity SIMPLE corrector
6. **Adjoint** pressure-velocity SIMPLE corrector
7. Update sensitivity, $\partial L / \partial \gamma$
8. Helmholtz PDE filter for $\partial L / \partial \gamma$
9. Update step coefficient, $\boldsymbol{\delta}$
10. Update design variable (OC-algorithm), $\boldsymbol{\gamma}$
11. Helmholtz PDE filter for $\boldsymbol{\gamma}$
12. Heaviside step projection for $\boldsymbol{\gamma}$
13. Converged? (No → Update inverse permeability, $\boldsymbol{\alpha}$)
14. Finish

Primal:

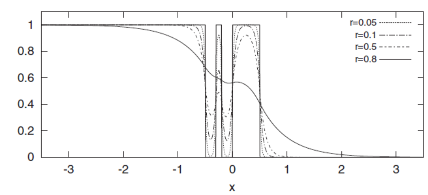
$$-\nabla \cdot \mathbf{u} \cong 0$$
$$(\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla \hat{p}_{rgh} - \nabla \cdot \big(2\nu D(\mathbf{u})\big) - g\rho_k + \alpha(\gamma)\mathbf{u} \cong 0$$
$$\mathbf{u} \cdot \nabla T - \nabla \cdot (K(\gamma)\nabla T) \cong 0$$

Adjoint:

$$-\nabla \cdot \mathbf{v} \cong 0$$
$$-(\nabla \mathbf{v})\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{v} + \nabla q - \nabla \cdot \big(2\nu D(\mathbf{v})\big) + \alpha(\gamma)\mathbf{v} + T_a \nabla T \cong 0$$
$$\mathbf{u} \cdot \nabla T_a - \nabla \cdot (K(\gamma)\nabla T_a) + \beta \mathbf{v} \cdot g + (T - T_{Obj}) \cong 0$$

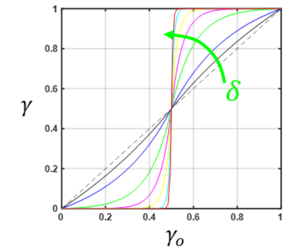
Helmholtz PDE filter:

$$-R_f \nabla^2 \phi + \phi = \phi_0$$



Heaviside step projection:

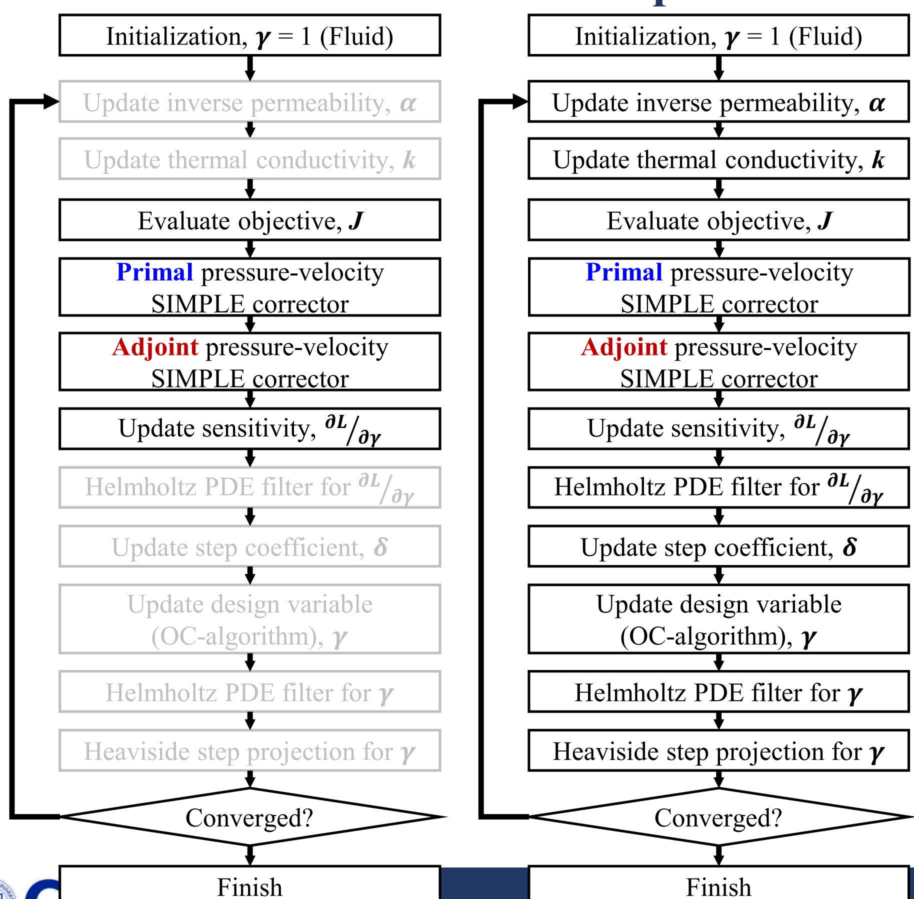
$$\gamma_i = 0.5\left[\exp\{-\delta(1 - 2\gamma_{o,i})\} - (1 - 2\gamma_{o,i})\exp(-\delta)\right] \quad [\gamma_i \le 0.5]$$
$$\gamma_i = 0.5 + 0.5\left[1 - \exp\left\{-\delta\left(\frac{\gamma_{o,i} - 0.5}{0.5}\right)\right\} + (\gamma_{o,i} - 0.5)\frac{\exp(-\delta)}{0.5}\right] \quad [\gamma_i > 0.5]$$

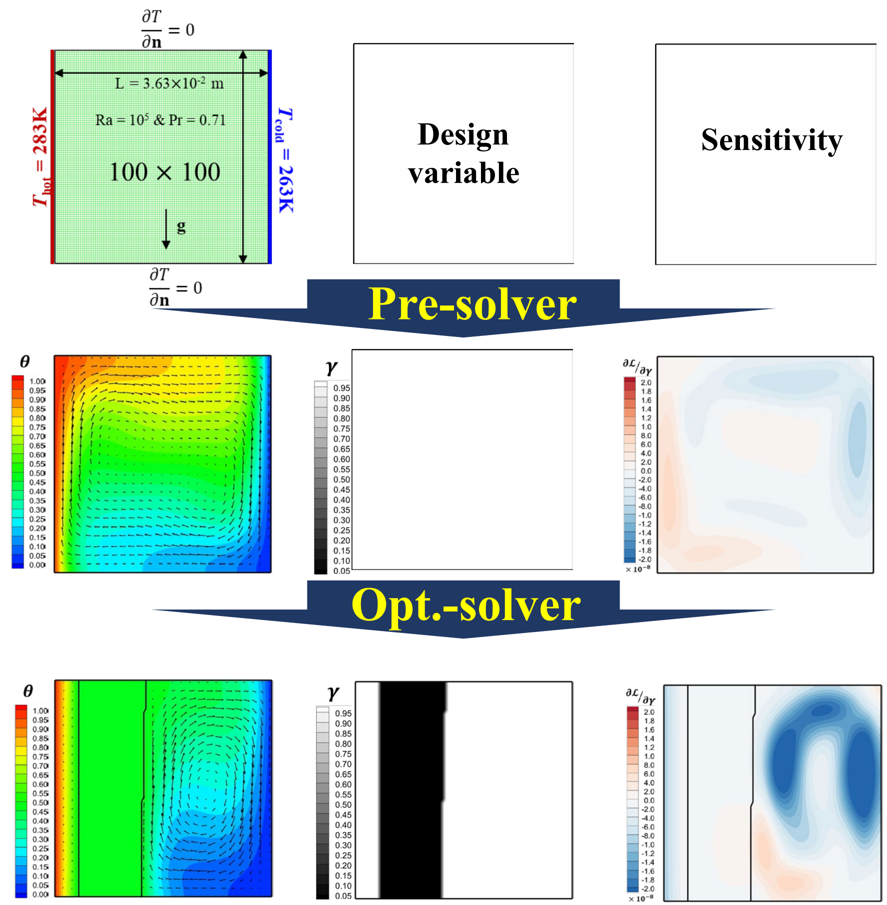


Numerical methodologies

❖ Strategy for natural convection case

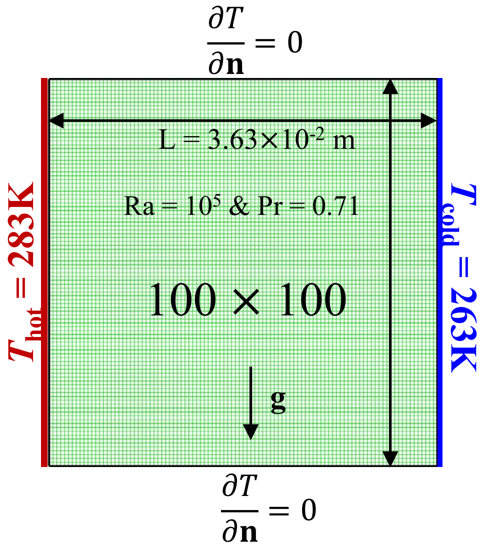
- The developed solver is separated into the pre-solver and optimization solver to obtain a stable solution, .
- The pre-solver gets a steady-state solution with sensitivity fields (w/o α and k update).
- The opt.-solver finds a topology optimization solution using an initial value for the steady solution of pre-solver.

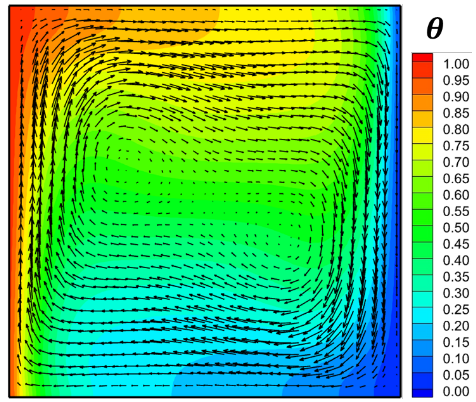




Numerical methodologies

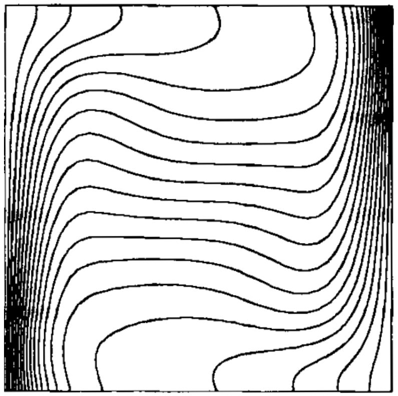
❖ Benchmark case [*Barakos et al., 1994]



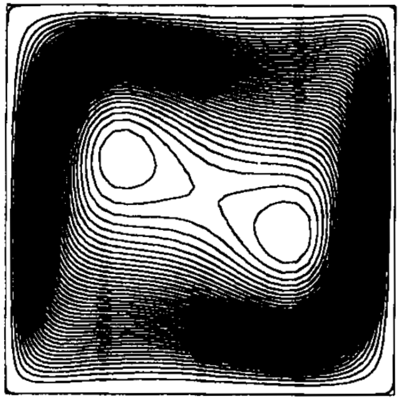


Steady solution

Barakos's results of Ra=10^5 case



Isotherm



Streamline

- The analysis domain and boundary conditions of previous study are considered to verify the developed solver.

❖ Definition of optimization problem

- **Minimize:** $J = \frac{1}{2}\int_{\Omega}\left(T - T_{Obj}\right)^2 d\Omega$

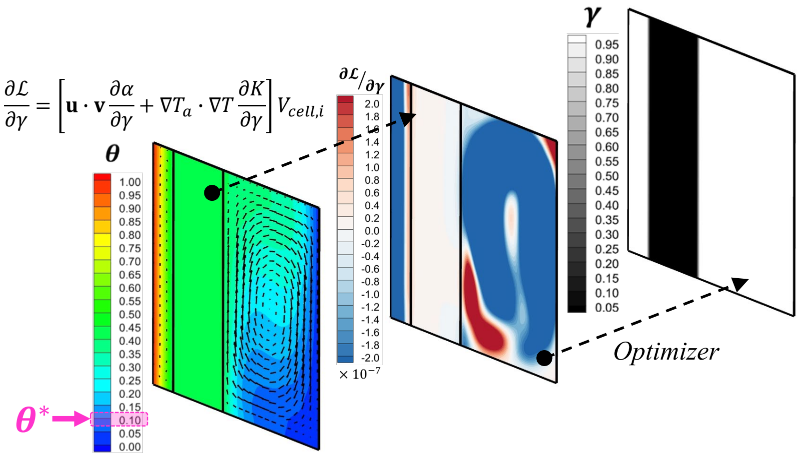
- **Subject to**

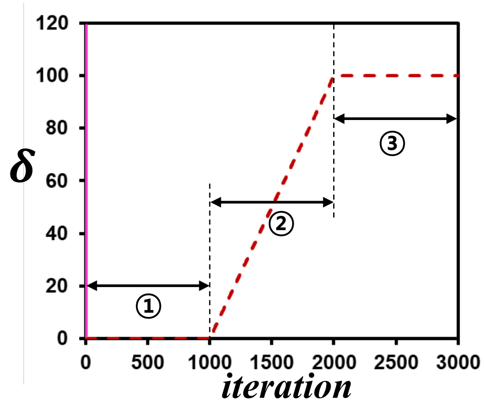
$$\begin{cases} \int_{\Omega} \gamma \, d\Omega - |\Omega|\psi = 0 \\ \Re_{\mathbf{u}}, \Re_p, \Re_T \cong 0 \\ \Re_{\mathbf{v}}, \Re_q, \Re_{T_a} \cong 0 \\ 0 \le \gamma \le 1 \end{cases}$$

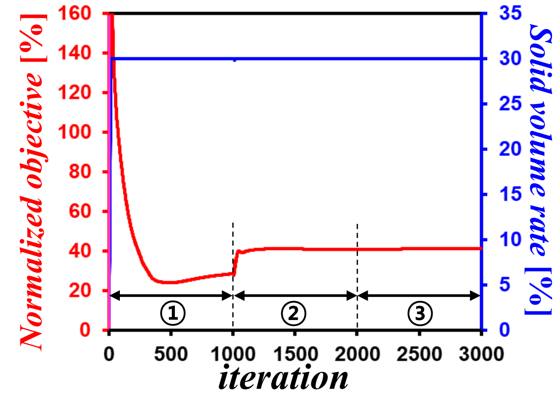
$$Normalized\ objective\ [\%] = \frac{Objective_i}{Objective_0} \times 100$$

❖ Optimization procedure

** Desired temperature, $\theta^* = 0.1$*
** Solid volume constraint, $\psi = 30\%$*







- The optimized configuration was obtained by a sequential procedure from thermal-fluidic fields to design variables distribution via sensitivity satisfying the solid volume constraint.
- ① **Iteration 0 – 1000:** Allow the grey zone to find a rough optimal shape.
- ② **Iteration 1000 – 2000:** δ increased to find the sharp interface.
- ③ **Iteration 2000 – :** Solution convergence for optimal configuration.

Results

R_f: Length parameter of Helmholtz PDE filter
Δx: Grid size
ψ: Solid volume constraint

❖ ELU function for coefficient of Heaviside step function

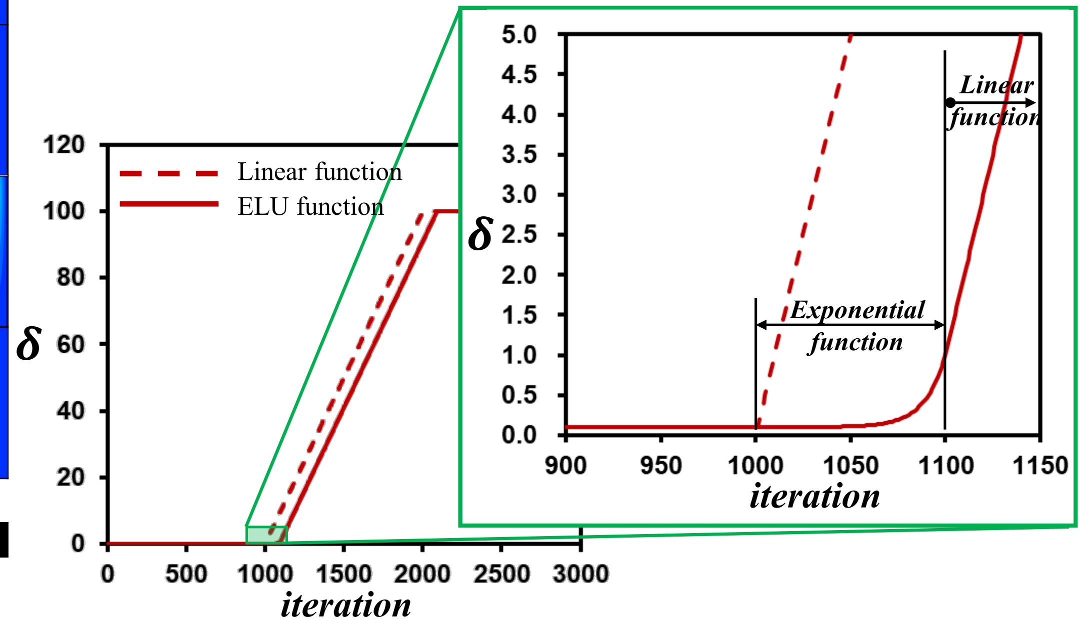
- Due to the rapid coefficient increase of a linear function, the distribution of design variables can vanish and doesn't match the solid volume constraint. → Require a gradual rise in coefficient
- ELU (Exponential Linear Unit) function instead of linear function for coefficient helps to find stable optimal solution.

$R_f/\Delta x$	ψ	Linear		ELU	
2	30%	30%	41.8%	30%	39.8%
	80%	87%	18.0%	85%	20.4%
4	30%	3%	101.1%	30%	48.6%
	80%	80%	25.8%	80%	25.8%

*Solid volume rate
*Normalized objective

※ Exponential Linear Unit (ELU)

$$\begin{cases} \delta_c = 0.1 & (i \le 1000) \\ \delta_E = 0.0001\left[e^{0.091051(i-1000)} - 1\right] + 0.1 & (1000 < i \le 1100) \\ \delta_L = 0.0999(i - 900) + \delta_E\big|_{i=1100} & (1100 < i) \end{cases}$$

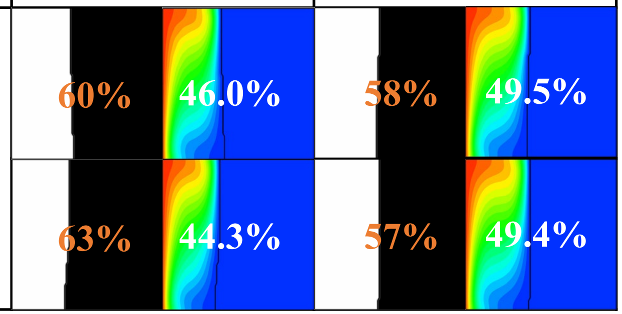


Results

R_f: Length parameter of Helmholtz PDE filter
Δx: Grid size
ψ: Solid volume constraint

❖ Harsh conditions to get an optimized solution

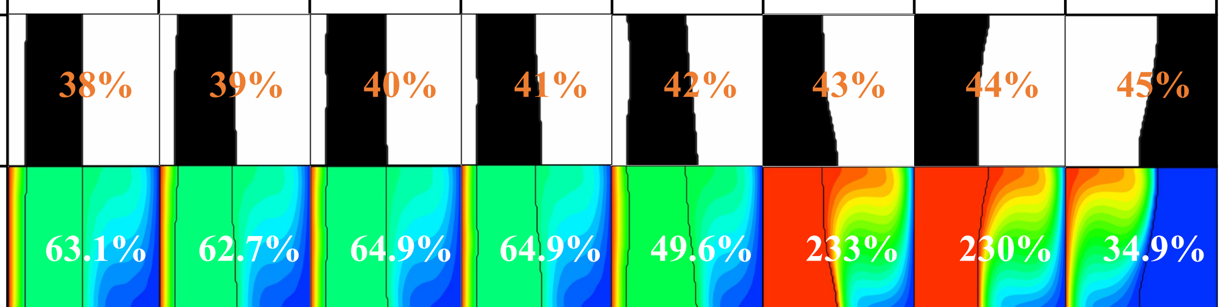
- **Cases of $\psi = 50\%$**
  - It is difficult to match the intermediate value of solid volume constraint.



$R_f/\Delta x$	ψ	Linear		ELU	
2	50%	60%	46.0%	58%	49.5%
4		63%	44.3%	57%	49.4%

*Solid volume rate
*Normalized objective

- **Transition point of optimized shape, $\psi = 42\sim45\%$**
  - Finding optimal solution for transition point from an air-insulation trend to a high diffusivity trend is challenging.



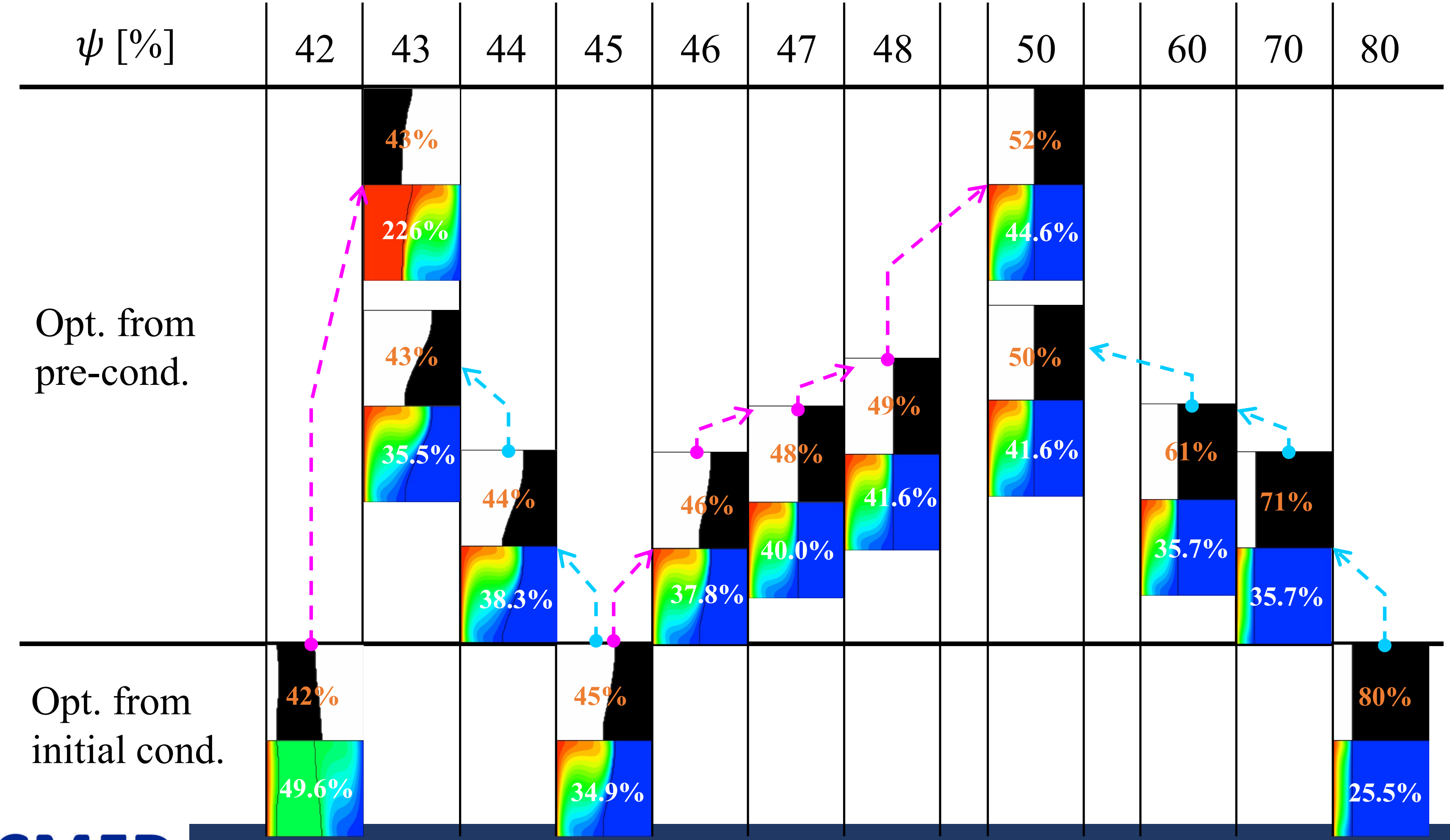
ψ [%]	38	39	40	41	42	43	44	45
γ	38%	39%	40%	41%	42%	43%	44%	45%
θ	63.1%	62.7%	64.9%	64.9%	49.6%	233%	230%	34.9%

*Solid volume rate
*Normalized objective

ψ: Solid volume constraint

❖ Effect of initial γ-distribution

- The stable optimal solution can be obtained when the previous results of solid volume constraints, which obtain the optimal solution, as the initial value.



Conclusion & Future works

- A topology optimization solver was developed based on the finite volume method of coupling *bouyantBoussinesqSimpleFoam* with *adjointShapeOptimizationFoam*.

- Helmholtz PDE filter and Heaviside step projection were considered to obtain a distinct optimal geometry.

- ELU function for the coefficient of Heaviside step projection leads to stable optimal solution.

- Additional study for start point of ELU function is required.

- It is recommended that the optimal solution close to the harsh condition sets as the initial conditions to obtain an optimal solution for harsh conditions.

- The effect of various conditions, such as solid volume constraints, conductivities, length parameters, desired temperature, etc., should be considered.

Thank you!

본 연구성과는 2024년도 정부(교육부)의 재원으로 한국연구재단의 지원을 받아 수행된 기초연구사업임(No. RS-2024-00449088)
This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education (No. RS-2024-00449088)